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SEDIMENT-LADEN TURBULENT FLOW: A REVIEW OF THEORIES AND MODELS

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SEDIMENT-LADEN TURBULENT FLOW: A REVIEW OF THEORIES AND MODELS

INTRODUCTION

This report presents a review of the theories which lie at the basis of current modelling approaches to sediment-laden turbulent free-surface flows. It originates from the literature study carried out in the framework of the FWO funded research by the author and as preparation for a presentation at the ERCOFTAC Belgian Pilot Centre Seminar on “New Developments and Validation in Turbulence Modelling” on 5 December 1997 in Ghent (Toorman, 1997). Throughout the project the draft version of this text has been improved.

The first useful advance in the study of the sediment-laden turbulent flows goes back to the early 30s, when von Karman (1934a) proposed his theories. A state-of-the-art on the subject at the time was given by O'Brien (1933). Vanoni (1944) summarized the following ten years of research. Much of the work is based on the study of equilibrium profiles in uniform open-channel flows. This led to the introduction of the well known Rouse profiles (Rouse, 1937), with many variants following in later years.

As these models rely on a mixing length hypothesis which is a function of the distance from the bottom, these models cannot be applied to general flow problems. In the early eighties the two-equation k-ε turbulence was first applied to sediment transport (DeVantier & Larock, 1983).

Despite these numerical tools, little progress has been made in improving the predictive quality of applied models, particularly when relatively high concentrations are involved. Several gaps remain in the understanding of the physics of the interaction of suspended sediment with turbulence and its modelling. Some of the latest findings relate coherent structures in turbulence to turbulent sediment dispersion. This has been reviewed by Nezu & Nakagawa (1993).

EQUILIBRIUM CONCENTRATION PROFILES

Velocity profile

The early studies of sediment-laden turbulent open-channel flow are based on the assumption of the validity of the logarithmic velocity profile, as expressed by the law of the wall, often written in a non-dimensional form as:

$$u_+ = \frac{1}{\kappa} \ln y_+ + A \tag{1}$$

where: $u_+ = u/u_*$ with u = the Reynolds-averaged velocity and u_* = the shear or friction velocity, defined as:

$$u_* = \sqrt{\tau_b / \rho} \tag{2}$$

with τ_b = the bed shear stress and ρ = the bulk density of the fluid (in our case, the suspension); y_+ = the non-dimensional coordinate, defined as:

$$y_+ = \nu y / u_* \tag{3}$$

with y = the distance from the bed (or vertical coordinate) and ν = the kinematic viscosity of the water; κ = the von Karman constant, A = an integration constant related to the bottom roughness. Some authors prefer to avoid the determination of A by using the velocity defect law (von Karman, 1934b):

$$\frac{u - u_{\max}}{u_*} = \frac{1}{\kappa} \ln \frac{y}{h} \tag{4}$$

where: h = the water depth (more general: the distance from the wall where $u = u_{\max}$).

Mixing length

The relationship between eddy diffusivity (or the turbulent momentum transfer coefficient) ν_t and the mixing length l, as proposed by Prandtl (1925), is given by:

$$\nu_t = l^2 \left| \frac{\partial u}{\partial y} \right| \tag{5}$$

Applying the mixing length theory, the rate of sediment mass transfer per unit area (G) is written, in analogy to that of momentum transfer, as (Vanoni, 1944; Rodi, 1980):

$$-\overline{u_i C} = -\beta\, v' l \frac{\partial C}{\partial y} = -\varepsilon_s \frac{\partial C}{\partial y} \tag{6}$$

where: v' = average of the absolute values of the fluctuations normal to the main flow, C = sediment mass concentration, β = empirical coefficient, ε_s = sediment transfer coefficient (or eddy diffusivity).

Equilibrium condition

The sediment mass conservation (or transport) equation, expressed in cartesian coordinates for a 2DV plane, is generally of the following form:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{\partial}{\partial x}\left(\varepsilon_{sx}\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(w_s C + \varepsilon_{sy}\frac{\partial C}{\partial y}\right) \qquad \textbf{(7)}$$

where: u = (Reynolds-averaged) horizontal flow velocity, v = (Reynolds-averaged) vertical flow velocity, w_s = sediment settling velocity, x = horizontal coordinate, t = time. In the context of open-channel flow, y = distance from the bottom. Horizontal and vertical mass diffusivity are often assumed to be equal[1], assuming isotropy, i.e. $\varepsilon_{sx}=\varepsilon_{sy}=\varepsilon_s$. In the case of steady open-channel flow over a flat bottom, eq.(7) reduces to the following vertical equilibrium:

$$w_s C + \varepsilon_s \frac{\partial C}{\partial y} = 0 \qquad \textbf{(8)}$$

This equation is sometimes referred to as the Schmidt (1925) equation. Integration of eq.(8) yields:

$$\ln\frac{C}{C_a} = -\int_a^y \frac{w_s}{\varepsilon_s}\,\mathrm{d}y \qquad \textbf{(9)}$$

where: a = arbitrary reference level.

In a uniform open-channel where the width to depth ratio is large (allowing secondary flow effects to be neglected), the x-momentum equation reduces to:

$$\frac{\partial \tau}{\partial y} = \frac{\partial}{\partial y}\left(\rho(\nu+\nu_t)\,\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} \qquad \textbf{(10)}$$

where: ν = the kinematic viscosity of the suspension, ν_t = the eddy viscosity, p = the fluid pressure. As the horizontal pressure gradient in uniform flow is constant, - $\partial p/\partial x = \mathrm{d}p/\mathrm{d}x = A$, integration of eq.(10) shows that the shear stress is a linear function of depth:

$$\tau = \rho(\nu+\nu_t)\,\frac{\partial u}{\partial y} = A(h-y) = \tau_b(1-y/h) \qquad \textbf{(11)}$$

where: τ = the shear stress. Knowing the velocity gradient, which can be computed from the velocity profile, eq.(1) or eq.(4), assuming κ to be constant, i.e.:

$$\frac{\partial u}{\partial y} = \frac{u_*}{\kappa y} \qquad \textbf{(12)}$$

and neglecting the molecular viscosity, one finds a parabolic profile of the eddy viscosity:

$$\nu_t = \kappa\, u_* y\,(1-y/h) \qquad \textbf{(13)}$$

[1] For 3D applications, the horizontal grid size may be much larger than the vertical one, requiring another mixing coefficient which also accounts for the large eddies.

Concentration Profiles

Schmidt profiles

In the earliest theories, the settling velocity is assumed to be constant. Schmidt (1925), in the context of dust transport in the atmosphere, assumed the mass diffusivity to be constant as well. Equation (9) then yields an exponential concentration profile:

$$\ln\frac{C}{C_a} = -\frac{w_s}{\varepsilon_s}\,(y-a) \qquad \textbf{(14)}$$

Derivation of the original Rouse profile

In general, ε_s is a function of y. If one assumes that $\varepsilon_s = \nu_t$, and using the mixing length theory, one can write:

$$\ln\frac{C}{C_a} = -\int_a^y \frac{w_s}{l^2\dfrac{\partial u}{\partial y}}\,\mathrm{d}y = -\int_a^y \frac{\rho w_s}{\tau}\frac{\partial u}{\partial y}\,\mathrm{d}y \qquad \textbf{(15)}$$

The mixing length for sediment-free open-channel flow is commonly expressed in terms of distance from the bed, and can be found by combining eq.(5), (12) and (13), giving:

$$l = \kappa y(1-y/h)^{1/2} \qquad \textbf{(16)}$$

It is is assumed in this approximation that w_s and ρ (= ρ_w) are constant. Introduction of eqs. (12) and (16) into (15) and integration yields:

$$\frac{C}{C_a} = \left(\frac{h/y-1}{h/a-1}\right)^Z \qquad \textbf{(17)}$$

where: $Z = w_s/\kappa\, u_*$, known as the Rouse number. This concentration profile was first derived by Ippen in 1936 at the suggestion by von Karman, but named after Rouse, who published it in 1937. Rouse profiles have successfully been validated by various experimental data (e.g. Rouse, 1938; Vanoni, 1944). But not all data can be matched ...

Generalized Rouse profiles

Since Rouse, many alternative profile equations have been proposed to provide corrections for deviations that are observed. Analysis of these profiles shows that many are of the same form (Ni & Hui, 1988). Many of them can be generalized (Ni, 1989; Ni & Wang, 1991) by taking away the restriction of the exponent 1 of the factor (1-y/h) in eq.(13) and by implementing a concentration dependence of the settling velocity, such as the Richardson-Zaki relationship:

$$w_s = w_0(1-\varphi)^\alpha \qquad \textbf{(18)}$$

where: φ = volumetric concentration. The generalized Rouse profile then is the solution of:

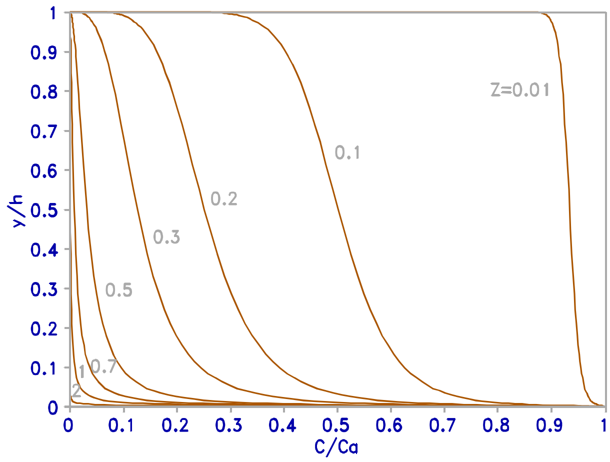

Figure 1: Rouse profiles for $Z = 0.1, 0.2, 0.3, 0.5, 0.7, 1, 2$ and 5.

$$\ln\left(\frac{C}{C_a}\right) = -\int_a^y \frac{w_s}{l^2 \frac{\partial u}{\partial y}}\, dy = -\frac{w_0}{\kappa u_*}\int_{\eta_a}^{\eta} \frac{(1-\varphi(\eta))^{\alpha}}{\eta(1-\eta)^{2n}}\, d\eta \qquad \textbf{(19)}$$

where: η = the non-dimensional coordinate y/h.

It can be demonstrated that the mixing length hypothesis of Hunt (1954), derived from von Karman's similarity assumption:

$$l = \kappa \frac{\frac{\partial u}{\partial y}}{\frac{\partial^2 u}{\partial y^2}} = 2\kappa h\sqrt{1-y/h}\left(b-\sqrt{1-y/h}\right) \qquad \textbf{(20)}$$

is equivalent to the assumption $n = 0.8$ (Ni, 1989). Hunt's profile (where $\alpha = 1$) is written as:

$$\frac{C}{C_a}\,\frac{1-C_a}{1-C} = \left(\sqrt{\frac{1-y/h}{1-a/h}}\left(\frac{b-\sqrt{1-a/h}}{b-\sqrt{1-y/h}}\right)\right)^Z \qquad \textbf{(21)}$$

where: $0.995 < b < 1$ and $Z = w_s/\kappa b u_*$.

Data analysis led Umeyama & Gerritsen (1992) to define n as:

$$n = \frac{1}{2}\left(1 + \beta\frac{C}{C_a}\right) \qquad \textbf{(22)}$$

With this formulation Umeyama (1992) succeeded to reproduce experimental data (i.e. Vanoni and Einstein & Chien).

Variants and extensions

Ippen (1971) proposed an alternative empirical velocity profile:

$$\frac{u - u_{max}}{u_*} = \frac{1}{\kappa'}\ln\left(\frac{y}{h} - \psi\ln\frac{y}{h}\right) \tag{23}$$

Itakura & Kishi (1980) derive the following velocity defect law (based on the Monin-Obukhov theory for atmospheric turbulence):

$$\frac{u - u_{max}}{u^*} = \frac{1}{\kappa}\left(\ln\frac{y}{h} - \alpha\frac{h}{L}(1 - y/h)\right) \tag{24}$$

where: α = Monin-Obukhov coefficient; L is the Monin-Obukhov length scale, a characteristic length, defined as:

$$L = \frac{u_*^3}{\kappa g(\gamma - 1)w_s\varphi} \tag{25}$$

The ratio y/L can be shown to be equivalent to the flux Richardson number (Toorman, 2000a). The mass diffusivity then becomes:

$$\varepsilon_s = \kappa u^* y\frac{(1 - y/h)}{(1 + \alpha y/L)} \tag{26}$$

which results in the following concentration profile:

$$\frac{C}{C_a} = \left(\frac{a}{y}\left(\frac{h - y}{h - a}\right)^{1+\beta}\right)^z \tag{27}$$

where: $\beta = \alpha h/L$. For $\beta = 0$ the Rouse profiles are found.

Other attempts have been proposed. A critical evaluation is necessary, as is illustrated by the following case. The second-order equation, proposed by Cao *et al.* (1995) (their eq.25) is wrong, as has been rightfully addressed in a discussion on the paper by Julien (1997). The error is most clear in their eq.(23), which implies a non-zero flux at the bottom (i.e. no equilibrium !). The actual source of error is the incorrectness of their eq.(16).

Avoiding the bottom singularity

A disadvantage of the Rouse profile is the singularity of the profile at the bottom ($C = \infty$ at $z = 0$) due to the use of the traditional logarithmic profile. This can be avoided by using the modified log-law proposed by Christensen (1972), which yields a zero velocity at the bottom:

$$u = \frac{u_*}{\kappa}\ln(1 + y/y_0) \tag{28}$$

with y_0 a measure of the roughness height. The eddy viscosity becomes:

$$\nu_t = \kappa u_* \left(y + y_0\right) \left(1 - y/h\right) \tag{29}$$

which is *not* exactly zero at the bottom. The corresponding concentration profile is found as:

$$\frac{C}{C_0} = \left(\frac{1 - y/h}{1 + y/y_0} \right)^{Z/(1 + y_0/h)} \tag{30}$$

This profile corresponds well with the original Rouse profile, except near the bottom, when the reference level is taken as $a = y_0$.

This is actually equivalent to the approach of Willis (1986). He proposed an extension of the Rouse profiles by adding the viscous sublayer of thickness y_0. In the turbulent layer the Rouse profile must be rewritten as:

$$\frac{C}{C_0} = \left(\frac{1 - (y - y_0)/(h - y_0)}{1 + (y - y_0)/\alpha(h - y_0)} \right)^{z/(1 + \alpha)} \qquad (y > y_0) \tag{31}$$

where: $\alpha = \varepsilon_0/\kappa u_*(h - y)$; the index 0 refers to y_0. For the viscous sublayer a similar relationship is assumed:

$$\frac{C}{C_b} = \left(\frac{1 - y/h}{1 + y/\alpha h} \right)^{10z/(1 + \alpha)} \qquad (y < y_0) \tag{32}$$

where: C_b = bed concentration. Notice that the factor 10 in the exponent implies a reduction of the von Karman constant by this factor. He also gives a solution for a negative y_0, which gives a finite value of the turbulent mixing at the bed.

THE VON KARMAN COEFFICIENT

The von Karman "constant" is an empirical constant. The values for turbulent flow in smooth pipes by Nikuradse (1932), giving $\kappa = 0.4$ and $A = 5.5$, have been often used for open channel flow. Detailed laser-doppler measurements by Nezu & Rodi (1986) show that in clear water open-channel flow the log-law can be applied strictly to the near-wall region only with universal values of $\kappa = 0.412$ and the integration constant $A = 5.29$ (with a standard deviation < 0.17).

Theoretical approximation of the von Karman constant

The meaning of the von Karman constant is rarely presented. Ni & Wang (1991) present a nice analysis. Consider the equilibrium condition in sediment-laden open-channel flow, equation (8). The mass diffusivity is assumed to be of the form:

$$\varepsilon_s = \frac{1}{2} l_1 \overline{|v'|} \tag{33}$$

where: l_1 = characteristic length scale for the vertical sediment movement. It is assumed that the fluctuation of the velocity component, perpendicular to the flow direction, is normally distributed:

$$f(v') = \frac{1}{\sqrt{2\pi}\sigma_{v'}} \exp\left(-\frac{v'^2}{2\sigma_{v'}^2}\right) \tag{34}$$

where: $\sigma_{v'} = \sqrt{\overline{v'^2}}$. Hence, $f(|v'|) = 2f(v')$, and:

$$\overline{|v'|} = \int_0^\infty f(|v'|)\, v'\mathrm{d}v' = \sqrt{2/\pi}\,\sigma_{v'} \tag{35}$$

Substitution of eqs.(33) and (35) into eq.(8) yields:

$$\frac{\partial C}{\partial y} = -\frac{\sqrt{2\pi}\, w_s C}{u_* L} \tag{36}$$

with: $L = l_1\sigma_v/u_*$. A generalized expression is:

$$L = y\left(1 - \frac{y}{h}\right)^n \tag{37}$$

where the index n reflects the effect of the fluid and sediment properties on the characteristic length for sediment motion. Substitution of eq.(37) into (36) and integration yields:

$$\int_{C_a}^{C} \frac{\mathrm{d}C}{Cf(C)} = -\int_a^y \frac{\sqrt{2\pi}w_0}{u_* y(1 - y/h)^n}\,\mathrm{d}y \tag{38}$$

where: $f(C) = w_s/w_0$, the hindered settling function. For $f(C) = n = 1$ one finds back the Rouse profiles. Comparison shows that the theoretical value of the von Karman constant should be $\kappa = (2\pi)^{-1/2} = 0.399$. As this is about the same value experimentally found, the assumption of the normal distribution of the vertical fluctuations seems to be justified (at least, far enough from solid walls).

Experimental determination of the von Karman coefficient

The log-law for the velocity profile is tested by plotting the dimensionless distance from the bed (y) as a function of the dimensionless velocity u_+. The value of the von Karman coefficient κ is computed from the slope of the non-dimensionalized velocity profile on a semi-logarithmic plot.

Analysis of various data sets using these plots (e.g. Vanoni, 1944; Ismail, 1952; Einstein & Chien, 1955; Elata & Ippen, 1961; Wang, 1981; Wang & Qian, 1992), suggests that the von Karman coefficient for suspension flow is not a constant, but shows a decreasing trend with increasing concentration. Wang (1981) attempted to formulate an empirical equation for the determination of κ for various conditions of velocity and sediment concentration. However, its application is quite complex and is based solely on equilibrium profiles data. Ippen (1971) links the variation of κ to the change in viscosity of the suspension due to the present particles.

The hypothesis that κ is not constant has widely spread, even though some data show no change or even an increase. In the early eighties this was finally opposed. It was noticed e.g. by

Itakura & Kishi (1980) that the velocity profile deviates from the ideal log-law near the free surface. This effect is even more pronounced with suspended sediment. Both Fukuoka (1980) and Itakura & Kishi (1980) succeeded to reproduce various data even with a constant κ, by neglecting the surface data points.

Coleman (1981 & 1986) made a special effort to rehabilitate the von Karman coefficient as constant. He shows that κ can he held constant if one introduces a so-called wake function to correct for the outer flow (i.e. the near-surface or pelagic) region. The complete velocity profile over the boundary layer above the viscous sublayer is expressed as:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{u_* y}{\nu} + A + \frac{\Pi}{\kappa} \omega(y/\delta) \tag{39}$$

where: δ = the height of the inner region, Π = wake strength coefficient. The first two terms are the known terms of the universal law of the wall, with A = the boundary integration constant, including the velocity reduction of the boundary roughness (e.g. Hama, 1954) and of the suspended sediment. The last term is the so-called wake region velocity augmentation function, proposed by Coles (1956), in which ω is a functional symbol with conventionally established values ranging from 0 at $y = 0$ to 2 at $y = \delta$. Coles (1956) proposed the following approximating empirical equation:

$$\omega = 2 \sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right) \tag{40}$$

This term accounts for the deviation from the log-law. The corresponding velocity defect law then becomes:

$$\frac{u_{max} - u}{u_*} = \frac{1}{\kappa} \ln\left(\frac{y\nu_\delta}{\delta\nu}\right) + \frac{\Pi}{\kappa}\left(2 - \omega\left(\frac{y\nu_\delta}{\delta\nu}\right)\right) \tag{41}$$

with ν_δ the suspension viscosity at level δ. The wake strength coefficient can be computed as (Coleman, 1981):

$$\Pi = \frac{\kappa}{2}\left(\frac{u_{max} - u}{u_*}\right)_{y/\delta = 1} \tag{42}$$

Analysis of his own experiments, led Coleman to conclude that, since the ratio $\nu_\delta/\nu \approx 1$ for all his profiles, only the wake strength parameter Π is affected by the presence of suspended sediment. With this analysis he obtained a mean value of $\kappa = 0.43$ with a standard deviation of 0.03. A unique relationship is found between Π and the global Richardson number, defined as:

$$Ri_g = \frac{g\delta(\rho_\delta - \rho_0)}{\bar{\rho} u_*^2} \tag{43}$$

Coleman's velocity profile proposal has been "tested" successfully by new laser-doppler data for clear water by Nezu & Rodi (1986). They demonstrate that the eddy viscosity then becomes:

$$\nu_t = \frac{\kappa u_* y(1 - y/h)}{1 + \pi(y/h)\Pi\sin(\pi y/h)} \tag{44}$$

and the mixing length:

$$l = \frac{\kappa y\sqrt{1 - y/h}}{1 + \pi(y/h)\Pi\sin(\pi y/h)} \tag{45}$$

The experimental data of Nezu & Rodi (1986) also confirm that Π has a significant influence on the maximum value of the eddy viscosity.

Nevertheless, whether the von Karman coefficient is constant or is dependent on the sediment concentration, is still subject of discussion. Wang & Qian (1992) made a new analysis of theory and experimental data and return to the old conclusion of a varying κ. They derive a rather complex equation, accounting for the effect for suspended sediment on the bulk viscosity, for the grain interparticle stress (based on the data from Bagnold, 1954) and for the damping of turbulence. Their derivation includes so many unjustified assumptions and empirical parameters, that one can arrive at any conclusion.

Gust (1976) seems to be the first to study the validity of the law of the wake for dilute clay suspensions in sea water. His experiments indicated that the viscous sublayer was 2-5 times larger than in the case of non-cohesive sediment suspensions. This could not be explained by an increase of the bulk kinematic viscosity of the suspension. The deviation from the universal law of the wall is similar to that observed in certain polymer dispersions, and is attributed to drag reduction.

Discussion

If one carefully reconsiders the various experimental conditions, then one should not forget that certain unverified assumptions have implicitly been used by the investigators, claiming a varying κ. The most important one is the assumption that the equilibrium profile for sediment-laden flow, like for sediment-free water, is still logarithmic.

Then, first of all, one often has to discard data points from a surface layer affected by free-surface effects in narrow channels. Indeed, secondary currents, induced by wall friction, result in a shear stress at the free surface, resulting in a maximum velocity in the main flow direction below the surface. For this reason it is better to use eq.(1) to evaluate κ, as this formulation does not depend on the depth. The question remains which points in the inner region should be withheld for the analysis.

Secondly, the assumption that the roughness parameter A in equation (1), which is generally not know exactly, is constant, may be false, particularly when deposition is unavoidable at over-saturated conditions at higher concentrations.

All the above described analytical models for the equilibrium profiles assume a certain empirical mixing length model. All (!) models seem to be able to produce satisfactory results for most of the published data sets of non-cohesive sediment. This is not surprising since the concentrations in most cases are so small that the suspended sediment has little effect on the profiles. The major effect is a significant decrease of the bottom shear stress.

New analysis and numerical experiments by Toorman (2000a) provide an alternative explanation. From the consistent implementation of buoyancy damping effects into the mixing-length theory it is concluded that κ indeed decreases with increasing stratification. As the Richardson number decreases significantly near the bottom, the stratification effects hardly affect the von Karman coefficient, which explains why Coleman can claim a constant κ. However, one should notice how few points near the wall Coleman uses to support his claims. The deviations above can be “explained” by a wake effect, but it does not imply that this is the physical wake. It is used by Coleman as a tuning parameter to match experimental profiles. It is the author’s conviction that there are near-surface deviations of the velocity profiles due to secondary currents, which are generated due to side-wall friction. This can be verified by quasi-3D modelling of steady open-channel flow. Applying such a model to clear water and suspension flow would demonstrate that there remain important deviations due to stratification effects.

Another argument, which supports a non-constant von Karman coefficient, starts from the basic mixing-length theory, where the original von Karman constant has been introduced for the first time, as κ is the proportionally factor between mixing length ℓ and distance from the wall. However, experimental and DNS data clearly show that the relationship $\ell = \kappa y$ no longer holds when approaching the wall in the viscous sublayer. Instead κ becomes zero (!) at the wall, which is the background of the Van Driest correction factor (see Toorman, 2000c).

THE MASS TRANSFER COEFFICIENT

The turbulent transport of a certain parameter is often assumed to be proportional to the gradient of this parameter, in direct analogy to the turbulent momentum transport (Rodi, 1980), as is expressed for sediments by eq.(6). The Reynolds analogy between mass and momentum transfer suggests that the corresponding transfer coefficients are closely related, which is expressed by:

$$\sigma_t = \frac{\nu_t}{\varepsilon_s} \tag{46}$$

where: σ_t = the turbulent Schmidt number (Rodi, 1980). Consequently, in order to generalise the Rouse profile equation, one should multiply the exponent Z with σ_t (e.g. Celik & Rodi, 1987). This can only be done if σ_t is constant, which generally will not be the case.

However, in most models it is assumed that the sediment mass transfer and momentum transfer coefficients are equal, i.e. that the turbulent Schmidt number equals 1. For simulations of simple equilibrium concentration profiles in uniform flow, this assumption seems to produce qualitatively satisfactory results (e.g. Toorman, 1997). But for higher concentrations and more complex flows there are clearly problems.

Experimental determination

Attempts have been made to determine the sediment mass transfer coefficient experimentally (e.g. Matyukhin & Prokof'yev, 1966; Majumdar & Carstens, 1967; Jobson & Sayre, 1970).

Jobson & Sayre (1970) start from the knowledge that the shear stress in steady open-channel flow varies linearly, according eq.(11). The momentum transfer coefficient can then be computed from experimental velocity profiles with:

$$\nu_t = \frac{u_*^2(1-y/h)}{\dfrac{\partial u}{\partial y}} \tag{47}$$

The shear velocity in open-channel flow can be determined experimentally as:

$$u_* = \sqrt{ghs} \qquad \textbf{(48)}$$

where: s = bottom slope. If the complete velocity profile is logarithmic, according to the combination of eqs.(47) and (12), the momentum transfer coefficient has a parabolic profile, eq.(13). This is not the case in reality.

A similar procedure can be applied for the mass transfer coefficient, starting from the following equilibrium for open-channel flow with an upstream point source of concentration:

$$u\frac{\partial C}{\partial x} = \frac{\partial}{\partial y}\left(\varepsilon_s\frac{\partial C}{\partial y} + w_s C\right) \qquad \textbf{(49)}$$

Jobson & Sayre (1970) integrate this equation over y, starting from a certain arbitrary value:

$$\frac{\partial}{\partial x}\int_y^h uC\mathrm{d}y = -\varepsilon_s(y)\left(\frac{\partial C}{\partial y}\right)_y - w_s(y)\,C(y) \qquad \textbf{(50)}$$

bearing in mind that the velocity u is independent on x, and the right-hand-side, evaluated at the surface, is zero because of the zero flux boundary condition.

Experimental data for dye, for which the settling velocity is zero, injected at the surface of fully developed turbulent open-channel flow, have been computed with eq.(50), resulting in a nearly parabolic profile. They obtained a value of the depth-averaged turbulent Schmidt number of 0.97 (Jobson & Sayre, 1970).

The same procedure, applied to fine and coarse sediment, but now taking into account the settling velocity, did not produce unique results. The cloud of experimental data points only suggests that near the bottom the mass transfer coefficient is larger than the momentum transfer coefficient.

One could also analyse data from equilibrium profiles and calculate the mass transfer coefficient from:

$$\varepsilon_s = -\frac{w_s(C)C}{\dfrac{\mathrm{d}C}{\mathrm{d}y}} \qquad \textbf{(51)}$$

The above analysis requires the a priori knowledge of the variation of the settling velocity with concentration. For non-cohesive particles a relationship exists for the case of settling in quiescent water (Toorman, 1999). There are indications that the settling velocity is affected by turbulent flow. Jobson & Sayre (1970) measured a small increase of the settling velocity in turbulent flow (3-6% for coarse sediment and 40-65% for fine sediment), which they attributed to turbulence and grouping.

A theoretical approach

Deigaard (1991) investigates the relationship between ε_s and the settling velocity, applying the mixing length hypothesis.

For neutrally buoyant particles the turbulent exchange between a layer and its neighbouring layers is considered. The turbulent exchange of neutrally buoyant matter can be written as:

$$\overline{v'C'} = -\varepsilon_s \frac{\mathrm{d}C}{\mathrm{d}y} = q\left(C - \frac{\mathrm{d}C}{\mathrm{d}y}\frac{l}{2}\right) - q\left(C + \frac{\mathrm{d}C}{\mathrm{d}y}\frac{l}{2}\right) = -ql\frac{\mathrm{d}C}{\mathrm{d}y} \qquad \textbf{(52)}$$

where: q = average vertical fluid flux, $(l/2)(\mathrm{d}C/\mathrm{d}y) = \Delta C$ = change in concentration over a distance $l/2$, C - ΔC = average concentration in the upward flux, $C + \Delta C$ = average concentration in the downward flux. In this case $\varepsilon_s = v_t = ql$, i.e. the Schmidt number $\sigma_t = 1$.

In the case that particles settle, there is loss of sediment from the volume of water, moving upward from y-$l/2$ to y. This amount can be estimated as:

$$\alpha_2 l^2 \frac{l}{2\alpha w} w_s (C - \Delta C) \qquad \textbf{(53)}$$

where: w (= r.m.s.(v')) = water velocity in the vertical exchange (strange enough not related to q by Deigaard), assumed to be proportional to the friction velocity, i.e. $w = \alpha_1 u_*$, $\alpha_2\, l^2$ = a characteristic horizontal area, $l/2w$ = time to travel from y-$l/2$ to y. At the same time settling sediment moves in, resulting in a gain of:

$$\alpha_2 l^2 \frac{l}{2}\frac{w_s}{w}(C - \Delta C/2) \qquad \textbf{(54)}$$

The total sediment concentration that has travelled upward is then obtained as:

$$C_{up} = \frac{1}{\alpha_3} l^3 \left(\alpha_3 l^3 (C - \Delta C) - \alpha_2 l^2 \frac{l}{2}\frac{w_s}{w}\frac{\Delta C}{2}\right) = C - \left(1 - \frac{\alpha_2}{4\alpha_3}\frac{w_s}{w}\right)\Delta C \qquad \textbf{(55)}$$

A similar expression is found for the downward concentration. The resulting net sediment flux then becomes:

$$\varepsilon_s = \left(1 - \beta \frac{w_s}{u_*}\right) \nu_t \qquad \textbf{(56)}$$

where: $\beta = \alpha_2/4\alpha_1\alpha_3$. This equation leads to an expression of the mass exchange coefficient which is always smaller than the eddy viscosity.

In the previous approximation, the change in settling velocity with concentration has not been accounted for. Assuming that the settling velocity can be described by the Richardson-Zaki relationship, $w_s = w_0(1 - \varphi)^n$, the change of settling velocity over a distance $l/2$ can be approximated by:

$$\Delta w_s = \frac{l}{2}\frac{\mathrm{d}w_s}{\mathrm{d}y} = -\frac{n w_s}{\rho_s (1 - \varphi)}\,\frac{l}{2}\,\frac{\mathrm{d}C}{\mathrm{d}y} \qquad \textbf{(57)}$$

Later, Fredsøe & Deigaard (1992) considered, in addition to turbulent diffusion, the contribution of advective sediment exchange. This resulted in an approximate formulation, which implies an increase with the settling velocity.

Turbulent bursting

Cao *et al.* (1996) propose a mass transfer coefficient based on a relatively simple bursting theory. It brings the location of the maximum diffusivity a little below mid-depth, in qualitative accordance to the trend in measurements. Intercomparison of the model with that of Hunt (1954) shows no improvement.

Stratification and buoyancy

When the bulk density of the fluid shows density gradients, it is called stratified. In the case of sediment-laden flows, stratification automatically results from the opposing forces of turbulent mixing and gravity, *cf.* the Rouse profiles. For positive density gradients, the stratification is unstable, and vice versa.

Stratification in shear flows influences the stability of these flows. The first stability criterium for stratified flow was proposed by Richardson (1920) in the context of atmospheric turbulence production where a sharp vertical gradient in the entropy occurs. Stratification in shear flows usually is characterised by a non-dimensional number, obtained by the ratio of buoyancy flux to shear production.

For a flow with density stratification the following time scales are defined (Landahl & Mollo-Christensen, 1986): (1) the mean flow rotation period:

$$t_r = \left(\frac{\mathrm{d}\overline{U}}{\mathrm{d}y} \right)^{-1} \tag{58}$$

(2) the intrinsic oscillation period for the fluid:

$$t_s = \left(-\frac{g}{\rho} \frac{\mathrm{d}\overline{\rho}}{\mathrm{d}y} \right)^{-1/2} = N^{-1} \tag{59}$$

where: N = the Brunt-Vräisälä or intrinsic frequency. The square of the ratios of time scales is the Richardson number, i.e. $Ri = (t_r/t_s)^2$. Hence, the gradient Richardson number is defined as:

$$Ri = \frac{-\dfrac{g}{\rho} \dfrac{\partial \rho}{\partial y}}{\left(\dfrac{\partial u_j}{\partial x_j} \right)^2} \tag{60}$$

The vertical turbulent transport of momentum and mass is strongly influenced by buoyancy effects: unstable stratification produces turbulence, stable stratification damps turbulence (like at a free surface). Hence, eddy diffusivity and mass transfer are reduced by stable stratification (Rodi, 1980), but only where a vertical gradient of density is present.

The stratification effect on the eddy viscosity usually is empirically expressed as a function of Ri and should fulfil the constraints: $\varepsilon = \varepsilon_0$ for $Ri = 0$ and $\varepsilon = 0$ for $Ri = \infty$. A simple equation which satisfies these conditions has been proposed by Rossby & Montgomery (1935):

$$\varepsilon = \varepsilon_0 \left(1 + \beta Ri \right)^{-n} \tag{61}$$

From energy considerations they found $n = 1/2$. The value of $\beta = 10$ has been first determined on the basis of measurements over a snow field by Sverdrup (1936). A similar relationship as (61) has been proposed by Munk & Anderson (1948) for the mixing coefficient. They determined the corresponding values of β = 10/3 and n = 3/2 on the basis of constraints, which can be derived from the investigations of Jacobsen (1913) and Taylor (1921). Several empirical function for the Schmidt number have been proposed in the literature (e.g. Nunez Vaz, 1994).

Hence, in simple models, as considered here, the buoyancy influence is accounted for with empirical formulations of the form:

$$\nu_t = \nu_{t0}(1 + \beta Ri)^{\alpha} \qquad \textbf{(62)}$$

$$\varepsilon_s = \varepsilon_{s0}\left(1 + \beta_s Ri\right)^{\alpha_s} \qquad \textbf{(63)}$$

According to a review study by Delft Hydraulics (1974), the Munk-Anderson (1948) parameters (α = -0.5, β = 10, α_s = -1.5, β_s = 10/3) fits best the majority of experimental data.

Le Hir (1997) modifies the mixing length and eddy viscosity as follows:

$$l = l_0 \exp(-aRi) \qquad \textbf{(64)}$$

$$\nu_t = \nu_{t0} \exp(-bRi) \qquad \textbf{(65)}$$

Furthermore, *Ri* is constrained to a maximum of 5, because the damping becomes too strong with this formulation.

For negative *Ri*, the damping function is set to 1 in the Delft Hydraulics code TRISULA (Uittenbogaard *et al.*, 1992).

It should be emphasized that stratified turbulent flow has been mainly studied in the context of thermally induced density effects. In the case of suspended particles, not only buoyancy effects play a role, but possibly also interparticle collisions, especially at higher concentrations. Further discussion on the correction factors, known as damping functions, can be found in the final report on parameterisation (Toorman, 2000a).

Approximated Ri-profile

One could have a first approximate idea of the variation of *Ri* over depth from the theoretical generalized Rouse profiles. According to the definition of *Ri*:

$$Ri = -\frac{g}{\rho}\frac{\dfrac{\mathrm{d}\rho}{\mathrm{d}y}}{\left(\dfrac{\mathrm{d}u}{\mathrm{d}y}\right)^2} = \frac{-gh}{\left(\dfrac{\rho_w}{\Delta\rho_a} + \dfrac{C}{C_a}\right)}\frac{\dfrac{\mathrm{d}(C/C_a)}{\mathrm{d}\eta}}{\left(\dfrac{\mathrm{d}u}{\mathrm{d}\eta}\right)^2}$$

$$= \frac{gh}{\left(\dfrac{\rho_w}{\Delta\rho_a} + \dfrac{C}{C_a}\right)}\frac{\dfrac{Z}{\eta^2}\left(\dfrac{C}{C_a}\right)^{\frac{Z-1}{Z}}}{\left(\dfrac{u_*}{\kappa\eta}\right)^2} = \frac{\kappa^4 gh}{w_s^2}\frac{Z^3}{\left(\dfrac{\rho_w}{\Delta\rho_a} + \dfrac{C}{C_a}\right)}\left(\frac{C}{C_a}\right)^{\frac{Z-1}{Z}} \qquad \textbf{(66)}$$

Applying the definition of *Ri* to a generalized Rouse profile, one finds:

$$
\begin{aligned}
Ri &= -\frac{g}{\rho}\frac{\dfrac{\mathrm{d}\rho}{\mathrm{d}y}}{\left(\dfrac{\mathrm{d}u}{\mathrm{d}y}\right)^2} = \frac{-g\Delta\rho_s}{\left(\rho_w + \Delta\rho\right)}\frac{\dfrac{\mathrm{d}\varphi}{\mathrm{d}y}}{\left(\dfrac{\mathrm{d}u}{\mathrm{d}y}\right)^2} \\
&= \frac{g\Delta\rho_s}{\left(\rho_w + \Delta\rho\right)}\frac{\dfrac{w_0 f(\varphi)\,\varphi}{\kappa u_* l}}{\left(\dfrac{u_*}{\kappa y}\right)^2} = \frac{\kappa g w_0}{u_*^3}\frac{y}{\left(1 - y/h\right)^n}\frac{\Delta\rho_s\varphi\left(1-\varphi\right)^\alpha}{\left(\rho_w + \Delta\rho\right)} \qquad \textbf{(67)} \\
&= \frac{\kappa g h w_0}{u_*^3}\frac{\eta}{\left(1-\eta\right)^n}\frac{\Delta\rho\left(1-\varphi\right)^\alpha}{\rho_w + \Delta\rho}
\end{aligned}
$$

One can then define a reduced Richardson number Ri_* as:

$$
Ri_* = \frac{Ri}{\dfrac{\kappa g h w_0}{u_*^3}} = \frac{\eta}{\left(1-\eta\right)^n}\frac{\Delta\rho\,\left(1-\varphi\right)^\alpha}{\rho_w + \Delta\rho} \qquad \textbf{(68)}
$$

Figure 2 shows that Ri_* is constant over depth for $Z = 1$. Ri_* increases with distance from the bed y when $Z < 1$ and decreases only for $Z > 1$.

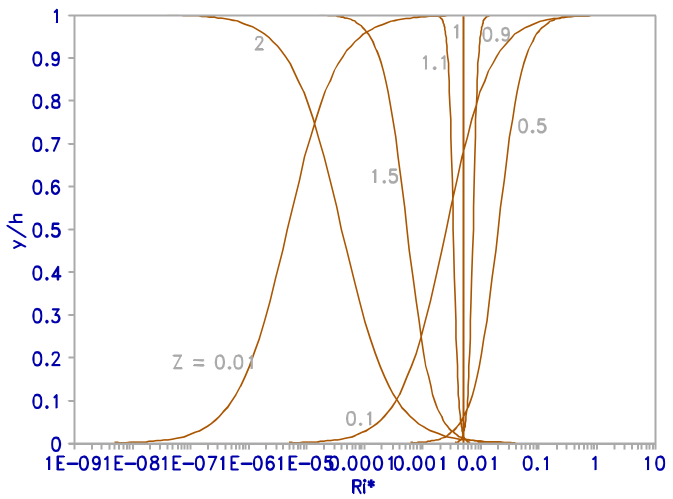

Figure 2: Variation of Ri_* over depth for various values of the Rouse parameter Z (for the parameter values: $\alpha = 0$, $n = 1$, $C_a = 8$ g/l, $\rho_s = 2650$ kg/m^3).

Extended model

The most refined analytical model for 1D profiles has been developed by Villaret & Towbridge (1991). They extend the above described model by including stratification effects (by making the eddy viscosity a linear function of the flux Richardson number), grain size distribution (assuming a narrow normal distribution around the median particle diameter) and Coles' wake correction (which has been discussed above). Several approximations have to be made in order to allow an analytical solution. The model has been validated with several data sets from the literature and a parameter sensitivity analysis has been carried out.

DRAG REDUCTION

It has been acknowledged from the first experimental data on that suspended sediment particles in turbulent flow change the velocity profile. Velocity gradients increase with sediment content due to the damping of turbulence by stable stratification, but the shear velocity decreases. The latter has the important consequence that for the same energetic conditions (e.g. the same pressure gradient), the velocity increases with increasing suspended load, resulting in much higher transport rates. This subject is studied in more depth in a separate report (Toorman, 2000b).

OSCILLATING GRID EXPERIMENTS

Stratification has been studied in oscillating grid tanks since the late 60s (e.g. Thomspon & Turner, 1975; E & Hopfinger, 1986; Wolanski *et al.*, 1989). The experimental procedure is more or less standardized. Nevertheless, not much is known about the mixing coefficient and its relation to turbulence characteristics. The majority of work has been done for thermal stratification. Quantitative results focussed on entrainment rates at the interface. Few experiments dealt with lutocline formation.

Most of these experiments have concentrated on the determination of a relationship for the entrainment rate v_E, which, based on dimensional arguments, can be written as a function of the Richardson number, and is found empirically to be of the form:

$$v_E = \frac{\mathrm{d}z}{\mathrm{d}t} = v_1 C_2 Ri^{-\beta} \tag{69}$$

where: z = layer depth, defined as the level of the interface minus the mean position of the grid; v_1 = r.m.s. value of the vertical component of the turbulent velocity, which can be fitted with an equation of the general form:

$$v_1 = C_1 \omega\, d^{\alpha+1} z^{-\alpha} \tag{70}$$

where: ω = frequency of the oscillation; d = representative length scale, characterized by the geometry of the grid and the stroke. Ri is defined (Huppert *et al.*, 1995) as:

$$Ri = g' \frac{l}{v_1^2} = g \frac{\Delta\rho}{\rho_0} \frac{l}{v_1^2} \tag{71}$$

where: l = integral length scale of the turbulence, g = gravity constant.

The value of the coefficient β in the literature varies between 6/5 and 7/4. The latter has been derived theoretically. A value of 3/2 has been found in many experiments, assuming $\alpha = 1$. Huppert *et al.* (1995) developed a method to derive both parameters and found $\alpha = 1.28 \pm 0.11$ and $\beta = 1.696 \pm 0.072$, which is close to 7/4.

In the analysis for suspensions, Huppert *et al.* (1995) assume a constant concentration over the dense layer. The final depth z_∞ was found to be a function only of α, total suspended mass and particle size. The effect of differential settling was not studied.

It is interesting to notice some analogy in concentration profile shape in oscillating grid tank experiments of E & Hopfinger (1987) and the Rouse profiles in equilibrium flow.

Numerical studies of sediment-laden flows

MIXING-LENGTH MODELS

The majority of engineering models for sediment transport use the mixing-length turbulence model with one or the other damping function (e.g. Le Hir, 1997, Cheviet *et al*, 2000). A basic description of the PML model can be found in e.g. (Toorman, 2000a).

Wolanski *et al.* (1988) succeeded to simulate lutocline formation in an estuary with a PML model. They solve the following mass balance equation:

$$\frac{\partial C}{\partial t} + w_s \frac{\partial C}{\partial z} = \frac{\partial}{\partial z}\left(\varepsilon_s \frac{\partial C}{\partial z} \right) \tag{72}$$

Note that this form is only valid for a constant settling velocity. The boundary condition at the free surface is a zero-flux Neuman condition and for the bottom, the flux is the net flux of erosion:

$$S(z=0) = A w_s \left((U/u_c)^2 - 1 \right) \qquad u > u_c \tag{73}$$

and deposition:

$$S(z=0) = w_s \left(1 - (U/u_c)^2 \right) \qquad u < u_c \tag{74}$$

where: U = depth-average current velocity and u_c = a critical velocity used as erosion threshold. It is assumed that the saturation concentration C_m = 10 g/l such that $A = 0$ when $C > C_m$, i.e. no further erosion and:

$$A = B\left(C_m - C(z=0) \right) \qquad C(z=0) < C_m \tag{75}$$

This artificial limitation to the maximum bottom concentration is explained by the likely breakdown of turbulent lift energy at high concentrations (i.e. the limit of turbulent CBS).

Based on the study of Armi (1978), where it is suggested that "the mixing may be predominantly bottom generated and limited to a bottom benthic boundary layer above which ε_s tends to zero", Wolanski *et al.* initially chose the mixing coefficient ε_s as:

$$\varepsilon_s = Dh/z \tag{76}$$

D is chosen such that the depth-averaged value equals to the depth-averaged value of the eddy diffusivity distribution corresponding the log-velocity profile, i.e. $D = 0.07u_*/\ln h$. Numerical problems near the bottom (infinite ε_s at $z = 0$) are avoided by keeping ε_s constant over the first 5 cm. But even this formulation leads to Rouse-like profiles. Therefore, they shifted to a ε_s dependent on the flux Richardson number and took the Munk-Anderson form:

$$K = F(z)(1 + 10Ri/3)^{-1.5} \tag{77}$$

where $F(z)$ is e.g. eq.(76).

The consistent implementation of damping functions in the PML model and its important implementations are discussed in (Toorman, 2000a).

THE *k*-ε TURBULENCE MODEL

The first tractable application of the *k-ε* model to suspended sediment transport has been carried out by DeVantier & Larock (1983), based on Rodi (1980b).

The standard *k*-ε turbulence model

The equation for transport of turbulent kinetic energy k is obtained by multiplying the Navier-Stokes equation with u and v respectively, giving a transport equation for the Reynolds stress. Averaging, making a summation and divide by two gives the exact form of the transport of k (e.g. Vandromme, 1993).

An equation for the transport of the energy dissipation rate ε is obtained by taking the derivative of the Navier-Stokes equations with respect to x_j, multiplying each side with $2\nu\, \partial u_i/\partial x_j$ and averaging (Launder, 1984) or by taking the curl of the u' equation, multiplying it by $\nabla\times u'$ and using the identity $\varepsilon = \nu\langle|\nabla\times u'|^2\rangle$ of homogeneous turbulence (Mohammadi & Pironneau, 1993). This derivation has been derived first by Davydov (1961).

The widely used, simplified standard form of the *k-ε* equations is:

$$\frac{\partial k}{\partial t} + U_i\frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + P + G - \varepsilon \qquad \textbf{(78)}$$

$$\frac{\partial \varepsilon}{\partial t} + U_i\frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) + c_1\frac{\varepsilon}{k}\big(P + (1-c_3)\,G\big) - c_2\frac{\varepsilon^2}{k} \qquad \textbf{(79)}$$

where: ν_t = the turbulent eddy viscosity, defined as:

$$\nu_t = c_\mu\, k^2/\varepsilon \qquad \textbf{(80)}$$

P = the production term, representing the flow induced turbulence production, defined as:

$$P = \nu_t D_{II}^2 \qquad \textbf{(81)}$$

where: D_{II} = shear rate intensity, the second invariant of the shear rate tensor:

$$D_{II}^2 = \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} \qquad \textbf{(82)}$$

G = the buoyancy term, which accounts for the damping of turbulent energy due stable stratification, defined as (DeVantier & Larock, 1983):

$$G = \frac{\overline{(\rho' - \rho_w)v'}}{\rho}g \qquad \textbf{(83)}$$

where: ρ_w = water density. In analogy with the eddy diffusivity concept, it is most commonly replaced by:

$$G = \frac{\beta g}{\rho} \frac{\nu_t}{\sigma_t} \frac{\partial \rho}{\partial y} \tag{84}$$

with: σ_t = the turbulent Prandtl-Schmidt number. Finally, c_μ, c_1, c_2, c_3, σ_k, σ_ε and β are empirical constants. The most commonly used set of values of these constants is (Rodi, 1980):

$$c_\mu = 0.09,\ c_1 = 1.44,\ c_2 = 1.92,\ \sigma_k = 1.0,\ \sigma_\varepsilon = 1.3.$$

Jacobsen (1989) and Mohammadi & Pironneau (1993) present the complete theoretical derivation of the values of the parameters c_μ, c_1 and c_2. The value of β depends on the choice of concentration variable; $\beta = 1$ when (excess) density is used.

The value of c_3 is still subject to discussion. The few references found give a wide variety. In order to be able to compare the various values, one has to look at the used definitions of the buoyancy term, its coefficients and of Ri_f. For simplicity, the new bulk buoyancy parameter c_G will be introduced as:

$$c_G = c_1 (1 - c_3) \tag{85}$$

The notation c_3 is withheld for the original coefficient as defined by Rodi (1980).

According to Rodi (1980) c_3 is expected to lie in the range 0-1. Hossain (1980) suggested $c_3 = 0.8$ ($c_G = 0.288$), based on the best simulation of experimental data.

Uittenbogaerd *et al.* (1992) presents a reasoning which leads to the conclusion that the Richardson number effect "is negligible for the scales where ε is important" (p.69), which then justifies the choice $c_3 = 1$ or $c_G = 0$, i.e. the buoyancy term in the ε equation is dropped.

Burchard & Baumert (1995) mention that Kochergin & Sklyar (1992) find the condition $c_{G\varepsilon} \geq 1$ (i.e. $c_3 < 0.306$). Baum & Caponi (1992) find theoretically $c_G = 1.14$ for thermal stratification, neglecting diffusion and assuming $Ri_T = 0.12$.

Jacobsen (1989) does not neglect diffusion. For the parameter c_G, stable stratified flow is considered. Validity of the logarithmic velocity profile is assumed. The flux Richardson number is defined here as $-G/(P+G)$, in correspondence with (Rodi, 1980). The k equation reduces to:

$$\varepsilon = P + G \tag{86}$$

This can be transformed to yield:

$$\varepsilon = \frac{\nu_T}{1 + Ri_f} \left(\frac{\partial U}{\partial y} \right) = \frac{1}{1 + Ri_f} \frac{u_*^3}{\kappa y} \tag{87}$$

Hence:

$$k = \frac{1}{\sqrt{1 + Ri_f}} \frac{u_*^2}{\sqrt{c_\mu}} \tag{88}$$

The ε equation becomes:

$$\frac{\partial}{\partial y}\left(\frac{\nu_T}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right) + c_1\frac{\varepsilon}{k}\left(P + (1-c_3)G\right) = c_2\frac{\varepsilon^2}{k} \qquad \textbf{(89)}$$

This can be rewritten to yield:

$$(1-c_3) = 1 + \frac{\sqrt{1+Ri_f} - 1}{Ri_f}\left(\frac{c2}{c_1} - 1\right) \approx 1.16 \qquad \textbf{(90)}$$

assuming that $Ri_f = 0.045$. Hence $c_G = 1.67$, which is quite different and gives an important contribution to the buoyancy term !

However, these positive values of c_G yield a negative contribution, i.e. a sink or decrease of ε due to buoyancy. This seems in contradiction with what is expected.

Burchard & Baumert (1995) determine the range of validity of applying the *k*-ε model, by neglecting diffusion and assuming constant forcing. They determine critical Ri_f numbers:

1) an underlimit for convection induced instabilities, known as Rayleigh-Bénard convection:

$$Ri_f \geq Ri_f^- = \frac{c_1 - 1}{c_G - 1} \qquad (1-c_3) < \frac{1}{c_1} \qquad \textbf{(91)}$$

which is based on a sufficient condition. However, the justification for this condition, in my opinion, is not well established.

2) an upper limit for Kelvin-Helmholtz instabilities:

$$Ri_f \leq Ri_f^+ \approx 0.25 \qquad \textbf{(92)}$$

which values has been derived theoretically. The steady state non-diffusive solution gives:

$$Ri_f^{eq} = \frac{c_2 - c_1}{c_2 - c_G} \geq Ri_f^- \qquad \textbf{(93)}$$

From these criteria follows that $c_G < 1$. Based on the numerical simulation of wind-induced thermal stratification, they finally propose a value $1\text{-}c_3 = -1.4$ which gives the smallest error between model and data.

Burchard & Baumert (1995) recognize the problem of other values, all positive (!), proposed elsewhere in the literature. For the present study of sediment-laden turbulent flow, data is not yet available to calibrate this parameter. Until then, it will remain an unresolved problem. It is only possible to carry out some sort of sensitivity analysis to see what the effect is of different values of c_3.

***Remark*: The buoyancy term**

Originally the buoyancy term in the ε equation was written as (*Rodi*, 1980):

$$\frac{\partial \varepsilon}{\partial t} + U_i\frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) + c_1\frac{\varepsilon}{k}(P+G)\left(1 + c_3 Ri_f\right) - c_2\frac{\varepsilon^2}{k} \qquad \textbf{(94)}$$

where: Ri_f = the flux Richardson number, which is usually defined as minus the ratio of buoyancy production of k to stress production:

$$Ri_f = -G/P \tag{95}$$

With this definition, some problems have arisen due to the fact that G was originally generalized to all dimensions. The resulting controversy regarding c_3 has been resolved by restricting G to the buoyancy production of the component, lateral to the flow direction (i.e. in our case, the vertical component) (Rodi, 1980). He proposes the following definition:

$$Ri_f = -\frac{G}{P+G} \tag{96}$$

which results in the above ε equation (64). Equation (64) is now generally used, but the chosen definition of Ri_f may still vary from one author to another. The background of the construction of the buoyancy term in eq.(89) is unknown to the author, but it seems that the factor $(1 + c_3 Ri_f)$ is only an empirical correction factor. Another choice of the definition of Ri_f than Rodi's may therefore affect the value of c_3.

Treatment of the free surface

The definition of proper boundary conditions for the free surface is a serious problem for the turbulence energy equations. Initially the free surface was treated in a similar way as a symmetry plane, i.e. the normal gradients of k and ε are assumed to be zero. The corresponding eddy viscosity distribution then shows a maximum at the free surface, as at the symmetry plane of fully developed turbulent flow between parallel plates. The few measurements, however, clearly show that the eddy viscosity becomes very small (nearly zero) at the free surface. Relatively little attention is given to this in the literature and a satisfactory general solution is not available. The first review of the problem is presented by Celik & Rodi (1984). An updated review is found in Section 4.4 of Nezu & Nakagawa (1993).

The presence of a free surface (or density interface) reduces the turbulence length scale (Rodi, 1980) as it suppresses the vertical movement of eddies (Nezu & Nakagawa, 1993). The vertical fluctuations are damped, while the horizontal ones are increased (Komori *et al.*, 1982; Celik & Rodi, 1984). This implies that k is not zero at the free surface and that the eddy diffusivity cannot be zero either. Measurements by Nezu & Rodi (1986) clearly show that k at the free surface is not zero indeed, which should be contributed to the presence of fluctuations in the plane of the surface.

Interesting experimental studies have been carried out by Ueda *et al.* (1977), Komori *et al.* (1982 & 1987), Nezu & Rodi (1986) and Kironoto & Graf (1994). The data show that the free surface affects only the layer above $y/h > 0.9$: a significant decrease of v'/u_* and a slight increase of u'/u_*. Also k/u_*^2 decreases, as the turbulence generation is much smaller than its dissipation.

An additional problem for the interpretation of the data is that most experimental channel flow data (e.g. Coleman, 1986) are biassed by secondary flow effects caused by the vertical channel walls, resulting in a reduction of the velocity towards the surface. This can be solved by including the wake function, as explained above.

The increasing instability of the free surface with increasing Froude number is normally neglected. However, the effect of waves contributes to an overall increase of the turbulent fluctuations (Nezu & Nakagawa, 1993).

Nezu & Nakagawa (1993) conclude that at present no correct boundary conditions for the turbulence equations can be defined. A few attempts have been made to define modified boundary conditions. Hossain (1980) proposes to apply a wall-like boundary condition for k:

$$k_s = u_s^2 / c_\mu^{1/2} \tag{97}$$

where: u_s = surface shear velocity. The value for ε at the surface is assumed to be related to k_s and a typical length scale of the shear layer, here the channel depth H, resulting in a modified wall-like condition:

$$\varepsilon_s = u_s^3 / a\kappa H \tag{98}$$

with: a = empirical constant. A disadvantage is that the value of the empirical constant is case dependent and that the definition of the depth is not always evident. Celik & Rodi (1984) apply a zero normal gradient condition for k (as for a symmetry plane) and:

$$\varepsilon_s = \frac{k_s^{3/2}}{aH} \tag{99}$$

Yacoub *et al.* (1992) suggested to prescribe the normal flux of ε at the free surface as:

$$\frac{\partial \varepsilon}{\partial z} = 3.5 \frac{\varepsilon^2}{k^{3/2}} \tag{100}$$

and zero gradients for the other variables.

In order to avoid the depth dependence of the length scale at the free surface, it is proposed by the present author to use the effective velocity gradient at the surface to compute the shear velocity and to use the distance to the next node under the surface as length scale. This gives acceptable results regarding the eddy viscosity profile.

Secondary flow effects

Many published data of fully-developed turbulent open-channel flow velocity profiles deviate from the theoretical log-law over a layer at the free surface. This layer is sometimes referred to as the outer region (Kironoto & Graf, 1994). The velocity profiles do not reach the maximum velocity at the surface, some distance below. The typical reduction at the free surface is known to be caused by secondary flow effects, i.e. vortices induced by friction with the side walls of the flume. This can be verified with quasi-3D models (e.g. Naot & Rodi, 1982). To allow the neglection of the counterworking secondary flow induced surface shear stress effect, a minimum width over depth ratio of 5 for the flume is required (Kironoto & Graf, 1994).

A possible approach to account for the secondary flow effects is the use of the wake function of Coles (1956).

Possibly, the free surface effect can only be accounted for correctly by solving the Q3D problem of the flow in the complete cross-section of the flume. A free surface correction only accounts for the free surface dissipation; not for the secondary flow induced shear stresses at the surface and near the bottom.

Whether the wake function is only related to secondary flow remains an open question. Otherwise, the log-law is invalid in the outer region even for infinitely wide channels.

Bottom boundary conditions for the sediment

In principle, the best boundary condition for the bottom is imposing the vertical sediment flux *S*, i.e. expressing the exchange of mass between bed and suspension. A positive (upward) flux means erosion, a negative (downward) flux means deposition.

In the case of unsteady flow, the erosion flux S_E can be expressed by the semi-empirical relationship attributed to Partheniades, but actually proposed by Ariathurai (1974):

$$S_E = M_E \max\left(\tau/\tau_E - 1\,, 0\right) \quad \textbf{(101)}$$

The factor $(\tau/\tau_E - 1)$ is a measure of the fraction of the particles at the bottom that are broken off from the bed surface, i.e. which become mobile and enter the computational domain.

The deposition flux S_D can be expressed by the semi-empirical relationship of Krone (1962):

$$S_D = -\max\left(1 - \tau/\tau_D, 0\right) w_s C \quad \textbf{(102)}$$

The factor $(1 - \tau/\tau_D)$ is a measure of the fraction of the particles at the bottom that do not stick to the bed, i.e. which remain mobile and should be incorporated into the computational domain. The total sediment flux *S* then becomes:

$$S = S_E + S_D \quad \textbf{(103)}$$

For the one-dimensional problem solved as steady state problem one should impose the bottom concentration to get a solution. In reality, this concentration is not known a priori.

Performance of the standard *k-ε* model

Equilibrium

Numerical models for sediment-laden flows are commonly validated against steady uniform sediment-laden open-channel flow data (e.g. DeVantier & Larock, 1983; Ouillon & Le Guennec, 1996). The most frequently used data sets are: Vanoni (1948), Einstein & Chen (1955), Coleman (1986) and Lyn (1987 & 1992). In general good agreement is found for the cases with small sediment concentrations.

For the higher concentrations, deviations become significant (velocities are overpredicted, the velocity gradients too small). The fact that the velocity gradients are underpredicted indicates that the damping is much stronger than can be explained by stratification alone. DeVantier & Larock (1983) rightfully suggest that a possible mechanism to explain this increased damping should be found in interparticle collisions. This indeed is a mechanism which is not accounted for thus far.

Interparticle collisions imply an increase of the shear stress. Consequently, the bulk fluid (i.e. the suspension) viscosity should no longer be taken constant (DeVantier & Larock, 1983), but it becomes a function of the concentration. However, the increase of the suspension viscosity with concentration is not significant enough and this modification alone turns out to be insufficient (confirmed by the author's experience). The important short-coming to the model is the fact that no additional damping term has been incorporated into the *k*-ε equations. See (Toorman, 2000a) for a detailed analysis.

Simulations with traditional damping function closures reveal problems due the growth of *Ri* to a very large value at the free surface. As the boundary conditions for concentration and velocity are zero gradient conditions, *Ri* tends to infinity at the free surface. Due to the limitations of the vertical discretization, the model gives a finite, but large value. This problem has been investigated within the COSINUS project. Its source and cure, are discussed in detail in (Toorman, 2000a).

Non-equilibrium

Non-equilibrium cases have been studied by Celik & Rodi (1985, 1988). Later studies include wave-induced sand transport over a rippled bed (Fredsøe, 1993; Trouw *et al.*, 1998) and cohesive sediment transport in estuaries (e.g. Cheviet *et al.*, 2000).

Present state

Since the first successful simulations with the k-ε model by DeVantier & Larock (1982), hardly any progress has been made. Celik & Rodi (1985, 1988) demonstrated that the k-ε model performs satisfactory for non-equilibrium conditions, which may have contributed to the lack of putting further effort in refining the model. Particularly, the problem of higher concentrations, especially for fine sediments, has not been resolved.

Actually, the numerical modelling of sediment-laden turbulent flows is still in an infant stage. One of the reasons is that few of the famous turbulence modelling research groups seem to have shown much interest into the matter. Celik & Rodi (1985, 1988 and 1991) did some work in this field. Most of the published work is done by less specialised engineers. Few have reached a stage past the modelling of the experiments of Coleman (1981, 1986) of simple equilibrium profiles (e.g. Ouillon & Le Guennec, 1996).

From own experience and what others say or do, it seems that for low concentrations equally good results can be obtained without the buoyancy terms. Consequently, Rouse-like profiles are still obtained. For instance, the buoyancy term in the k-ε equations is missing in the simulation of Celik & Rodi (1988). They use a constant value for the Schmidt number equal to 0.5, based on previous studies. Hossain & Rodi (1982) made an attempt to translate the *Ri*-dependence for thermally stratified flow to sediment-laden flows.

In Europe, the large commercial hydraulics laboratories, i.e. Danish Hydraulics, Delft Hydraulics, LNHE and HR Wallingford, have their own 3D sediment transport models. Delft Hydraulics DELFT-3D software package has been tested and used extensively in a reduced 1DV form, their "point-mud" model, for the SILTMAN project (Winterwerp & Uittenbogaard, 1997; Winterwerp, 1998).

Shortcomings

The Boussinesq approximation, assuming a linear relationship between the *Re*-stress tensor and the shear rate tensor, is not valid for several applications where sudden changes in mean strain rate occur, e.g. involving curved surfaces, 3D flows, boundary layer separation, rotating and, important within the present context, stratified flows (Wilcox, 1993). Non-isotropic turbulence is required, which cannot dealt with by the k-ε model.

The derivation of the standard k-ε equations assumes a constant fluid density. In analogy as for the transport of momentum, the k-ε equations may have to be extended with a term of the form $k/\rho\ \mathrm{d}\rho/\mathrm{d}t$ and $\varepsilon/\rho\ \mathrm{d}\rho/\mathrm{d}t$ respectively to account for additional density effects. But this is probably not the right way to incorporate these terms, as in reality Reynolds averaging should be applied.

ADVANCED MODELS

Recently, thanks to the increasing capacities of computers, more complicated and more detailed models have been applied to suspension flow problems.

Two-phase flow models (e.g. Viollet *et al.*, 1992; Crow *et al.*, 1996; Greimann *et al.*, 2000) require the explicit description of the interaction between the solid and the fluid phase. They can be applied to industrial internal flow problems.

Direct numerical simulation (DNS) models (e.g. Elghobahsi & Truesdell, 1993; Pan & Banerjee, 1997; Boivin *et al.*, 1998) are still restricted to small Reynolds numbers.

Galland *et al.* (1996) present results obtained with a Reynolds stress model. A major problem is the lack of data to calibrate the many parameters, particularly for application to sediment-laden flows (see also Toorman, 2000a).

In general, these models are too complicated to be applied to large-scale estuarine and coastal sediment transport problems. As research toll, they are very valuable. They provide new insights into the mechanisms of turbulence modulation. They may become useful to provide additional numerically generated data for the calibration and validation of the presently used applied models, based on mixture theory.

HIGH CONCENTRATION EFFECTS

Above a certain critical concentration, interparticle interactions are very important. These layers, which form at and occur on the bed surface can be termed most generally as concentrated benthic (i.e. near-bed) suspensions (CBS). In the case of non-cohesive sediments, they are known as sheet flows.

Concentrated benthic suspensions

Stable benthic suspensions with a sharp lutocline and concentrations up to 10 g/l can be found in nature. The most spectacular data set is presented by Wolanski *et al.* (1988). These layers are maintained by turbulent entrainment, which is in equilibrium with hindered settling.

CBS layers are distinguished from fluid mud layers by the flow regime, which is turbulent. CBS layers are maintained if there is enough energy for turbulent mixing. Mud layers are characterized by their high concentrations (of the order of 100 g/l) and their non-Newtonian, thixotropic behaviour. Their flow is restricted to the laminar regime.

Winterwerp (1996) argues that CBS layers of intermediate concentrations are inherently unstable: there is not enough turbulent energy to prevent particles from depositing. Within the COSINUS project, a detailed study of CBS has been carried out (Winterwerp *et al.*, 2000).

Sheet flow

At high shear stresses, bed forms disappear, the finer sediment fraction is transported in suspension and the coarser fraction is transported in a dense layer, close above the bed, with a thickness a several times the grain diameter (Fredsøe, 1993). The particles in this sheet flow layers are supported by forces associated to interparticle collisions, assumed to be similar to grain flow as described by Bagnold (1954). Most of the recent work on sheet flow has been carried out within the framework of the MAST I G6M and MAST II G8M Coastal Morphodynamics Research project.

The actual distinction between CBS and sheet flow probably originates from opposing net effects on the turbulent fluctuations: turbulence is damped in CSB while it is maintained and even increased in sheet flow. This can be attributed to the size effect, as studied by Gore & Crowe (1989): small particle cause damping, coarser particles cause production.

The mathematical modelling of sheet flows is "far from complete" (Fredsøe, 1993). A few simple analytical and more complex models are available (see Fredsøe, 1993, for an overview). The proposed models have in common that sheet flow is approached as bed load and the models consist of generalizations of Einstein's bed load formula.

Few experimental data are available, e.g. Willis (1986), Wilson (1988), Sumer *et al.* (1992) and Ribberink & Al-Salem (1992). Most of these experiments are for wave+current induced transport. Janssen (1995) has compiled a literature review.

Interparticle stress

Bagnold (1954 & 1956) made an experimental study of grain (or granular) flows. He concluded that this flow behaviour is characterized by two dimensionless constants. The grain shear-rate number is defined as:

$$N_s = \frac{\lambda^{1/2} \rho_s d^2}{\mu_w} \frac{\mathrm{d}u}{\mathrm{d}y} \qquad \textbf{(104)}$$

where: λ = the linear concentration, related to the volumetric concentration by:

$$\frac{\varphi}{\varphi_{max}} = \left(\frac{\lambda}{1 + \lambda} \right)^3 \tag{105}$$

The grain stress number is defined as:

$$G_s = \sqrt{\frac{\rho_s d^2 T}{\mu_w^2 \lambda}} \tag{106}$$

where T is either the grain shear stress or the "dispersive" pressure. Bagnold's theory assumes a constant concentration over the depth.

Chen (1988) demonstrates that the data from Bagnold and others can be simulated with a visco-plastic rheological model. As the latter is a laminar flow approach, the question is what flow regime Bagnold's data belong to.

Based on data from Bagnold (1954), Wang & Qian (1992) propose the following expression for the interparticle collision induced shear stress:

$$\tau_d = 0.04 \rho \lambda^{1.35} \left(D \frac{\partial u}{\partial y} \right)^2 \tag{107}$$

With $u_*/\kappa y = \partial u/\partial y$, eq.(107) becomes:

$$\tau_d = 0.04\, \lambda^{1.35} \left(\frac{D}{\kappa y} \right)^2 \tau_b \tag{108}$$

The proportionality with y^{-2} indicates that the grain shear stress becomes negligible away from the bed. According Einstein's postulate the upper boundary for bed load transport is at $y = 2d$ (d = particle diameter). Then one obtains $\tau_d = 0.0625\ \lambda^{1.35}\tau_b$. Note that here κ is assumed constant in the region of highest concentration. The authors conclude that τ decreases with concentration, which is an indication of drag reduction.

Dissipation by interparticle collisions

As collision increases the dissipation rate, possibly, an additional source term should be added to the RHS of the ε equation. The dissipation rate should increase with increasing sediment concentration as the collision frequency will increase. As it concerns a dissipation *rate*, for dimensional reasons, the source term should be written e.g. in terms of dC/dt. An increase of ε and a decrease of k with increasing concentration implies a reduction of the turbulent eddy viscosity, i.e. damping. This damping will also occur when the concentration distribution is homogeneous ($dC/dx_i = 0$), and is thus different from the buoyancy term, which is proportional to the concentration gradient. Turbulence modulation has been measured in pipe flows of particle-laden turbulent flows (Gore & Crowe, 1989, have collected and analysed many data). This subject has been studied in the field of powder technology (e.g. Senior & Grace, 1998). Within the context of high-concentrated sediment transport, this topic is neglected. It may be worth to include it in future research as it may help to understand turbulence damping better.

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